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ATMOSPHERIC EFFECTS ON SPACE OBJECT IMAGERY

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This project deals with the efficient computer simulation of atmospherically degraded light beams. Such simulations are expected to be of use in the computer modeling of the operation of many optical devices sensing atmospherically degraded light, e.g., telescopes and cameras. Two methods of generating random phase and log-amplitude fields entering a receiver aperture are investigated. The first method generates a set of random arrays and tailors them so as to have the requisite probability distribution and covariance. The

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second (Karhunen-Loeve) method generates a set of eigenfunctions having the correct covariance. The series is then summed using random coefficients to provide one member of the set of wavefronts.

Finally, to assure that the computer generated arrays represent the real atmosphere as accurately as possible the relationship between the random arrays and atmospherically degraded log-amplitude and phase fronts is explored.

ATMOSPHERIC EFFECTS ON SPACE OBJECT IMAGERY

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INTRODUCTION

This is the first semiannual technical report under Contract F30602-74-C-0130 entitled Atmospheric Effects on Space Object Imagery. The report covers efforts during the period January 15, 1974 to June 1 1974. One object of the program is to study techniques for the computer generation of random wavefronts simulating atmospherically degraded light beams. This material is intended to be useful in the computer simulation of restoration and predetection compensation of atmospherically degraded optical images, especially images of high flying objects such as satellites. A second objective is to study angle of arrival fluctuations and high altitude temperature spatial spectra. This work will be considered in a later report.

In the design and construction of equipment to overcome the effects of atmospheric turbulence, it is often a difficult procedure to check out the equipment in the environment for which it is intended. Large telescopes are often busy and ground-level propagation ranges are not always accessible. For these reasons it is desirable to simulate the operation of such devices digitally. In order to perform digital simulation, one must have as exact a representation of the turbulent atmosphere as possible. This report concerns efforts to provide efficient computer simulation of atmospherically degraded light beams as a first step in simulation of a complete system.

Some information is available for simulating atmospherically degraded wavefronts. It is believed that the phase variations follow approximately Gaussian statistics as do phase difference fluctuations [Clifford, 1971] [Bertolotti, 1968] and log-amplitude variations [Ochs, 1969], [Tatarski, 1971, p. 292]. It is further expected that the phase structure function will follow a +5/3 law with separation in the inertial subrange [Bouricius, 1970]. Further, predicted expressions for phase structure and correlation functions are known [Tatarski, 1971]. These items are sufficient to allow the generation of a set of randomly degraded wave fronts.

In the present contract the objectives are to simulate random optical fields using two approaches, a random matrix approach and the Karhunen-Loève series approach, and to examine the statistics of atmospherically degraded images using recently obtained stellar image data. The random matrix approach has been used by others [McGlamery, 1974], [Bradley, 1974] to generate random phase fronts, but the processes of generating the associated log-amplitude fluctuations and of simulating the fields for a light beam that has propagated downward along a vertical path have not been hitherto considered. Neither has the application of the Karhunen-Loève series to this problem been considered. During the past quarter our efforts have concentrated on setting up and examining matrices of random numbers to assure that they satisfy the statistical criteria. Work on the Karhunen-Loève approach is just starting. The work on the examination of stellar images has not yet started.

In the present report the work on the generation of random phase matrices will be described first, followed by a derivation of the scheme for relating phase and amplitude fluctuations. Finally a brief discussion of the Karhunen-Loève approach will follow.

The procedure for generating matrices of random numbers with the desired statistics is quite straightforward. The first step is to generate an ensemble of arrays of random numbers with normal distributions and uniform average spectra. The second step is to compute the spectrum of each individual array, tailor these spectra to the desired average functional dependence, and transform back to obtain random arrays with the desired probability density and covariance.

Some of the details of the process we used will now be presented. The first step was to obtain a set of arrays of uncorrelated random numbers. These random number arrays consisted of matrices of 64 x 64 elements which were (theoretically) samples from a Gaussian probability distribution. These elements were produced by a machine-specific random number generator.

To insure that there be no correlation effects between the elements of the array a random loading algorithm was employed. In this algorithm two independent random number generators were used to generate numbers between one and sixty-four. These random number tuples were used to choose a point in the array which was then filled by a third generator with Gaussian random numbers between -3 and +3. This process was continued until the array was something more than half-filled. The remaining cells were then filled sequentially.

The random loading scheme just described obviously has some degree of redundancy due to the possibility of selecting the same point in the array more than once, but in the interest of generating arrays in which there is no element interaction (correlation), this redundancy was felt worthwhile.

The arrays were then tested for the desired statistical properties. This was accomplished by grouping the arrays by fives and doing an analysis of variance with two-way classification on each group. This test assumes a model of the form shown in Eq. (1).

(1) $$x_{ij} = \mu + \alpha_i + \beta_j + \sigma_{ij}$$

where x_{ij} is the generic array element; μ is the mean; α_i, β_j, and σ_{ij} represent respectively row effects, column effects, and row-column interactions or correlations. Statistics were generated from the arrays enabling us to perform F-tests for significance of these effects. No such significant effects were found.

The next step was to obtain the spatial spectrum of the individual arrays and to adjust the power spectra to have the desired functional dependence. This was accomplished using the Fast Fourier Transform (FFT) technique. After the spectral components were found, they were put in phasor form giving the magnitude and phase of each component. The magnitude was then multiplied by the value of the desired phase spatial spectrum for that value spectral component. The phase spatial spectrum chosen was based on the Von Karman index spectrum with a ten meter outer scale. This was available from work performed previously but was normalized to coherence length, r_0, as discussed in Appendix B. An inner scale much smaller than the aperture grid size was assumed. The complex phase of each spectral component was saved for use later in generating the log-amplitude. The modified spectra were then transformed back to the spatial domain. The result was the desired set of random arrays with the proper phase correlation function and normal probability distribution function.

After the phase fronts were generated and checked statistically, the associated log-amplitude fluctuations were considered. The derivation of the procedure for relating the log-amplitude to the phase will now be presented. The discussion uses the well-known complex log-amplitude representation. We assume that the (random) phase of the light wave entering a receiving telescope is known, and that the region inside the telescope has no index fluctuations so that the free space wave equation applies, $\varepsilon_1(\overline{r}) = 0$. The complex log-amplitude $\psi(\overline{r})$ is, following standard procedure, divided into an unperturbed plane wave, $-jkz$, with constant amplitude, $+\ell_0$, and a correction, ψ_1, due to the random fluctuation as in Eqs. (2). The correction follows the differential equation in Eq. (3) [Tatarski, 1971, p. 223].

(2a) $$E(\overline{r}) = e^{\psi(\overline{r})}$$

(2b) $$\psi = \ell_o - jkz + \psi_1$$

(2c) $$\psi_1 = \ell + j\phi$$

(3) $$\frac{\partial^2\psi_1}{\partial x^2} + \frac{\partial^2\psi_1}{\partial y^2} - 2\ yk\ \frac{\partial\psi_1}{\partial z} + k^2\varepsilon_1(\overline{r}) = 0.$$

Using Eq. (2c) we can separate Eq. (3) into real and imaginary parts giving two equations relating $\ell_1(\overline{r})$ and $\phi_1(\overline{r})$. They are

(4a) $$\frac{\partial^2\ell}{\partial x^2} + \frac{\partial^2\ell}{\partial y^2} = 2\ k\ \frac{\partial\phi}{\partial z}$$

$$\text{(4b)} \qquad \frac{\partial^2 \ell}{\partial x^2} + \frac{\partial^2 \ell}{\partial y^2} = -2\,k\,\frac{\partial \ell}{\partial z} \ .$$

The two coupled differential equations relating the log-amplitude and phase are easily solved. Solutions are given in Eqs. (5).

$$\text{(5a)} \qquad \phi = \sum\sum \phi_{mn} \cos\left(\frac{2\pi mx}{a} + \frac{2\pi ny}{b} + \left((\frac{2\pi m}{a})^2 + (\frac{2\pi n}{b})^2\right)\frac{z}{2k} + \theta_{mn}\right)$$

$$\text{(5b)} \qquad \ell = \sum\sum \phi_{mn} \sin\left(\frac{2\pi mx}{a} + \frac{2\pi ny}{b} + \left((\frac{2\pi m}{a})^2 + (\frac{2\pi n}{b})^2\right)\frac{z}{2k} + \theta_{mn}\right)$$

They are in Fourier series form to facilitate matching to the computer generated random phase and log-amplitude arrays. We note that the relationship between the phase and log-amplitude expressions is especially simple; the only difference is that cosines are replaced by sines.

Equations (5) are more than sufficient for fitting the computer-generated solutions. The summations cover the range $-\infty<m,n<\infty$ while only the range $<m,n<\infty$ is required to fit a given random matrix; i.e., two independent solutions are possible. Two such solutions are giver in Eqs. (6) and (7). They differ by choice of the relations between the independent constants.

$$\text{(6a)} \qquad \theta_{mn} = -\ \theta_{-m,-n}$$

$$\text{(6b)} \qquad \phi_{m,n} = +\ \phi_{-m,-n}$$

$$\text{(7a)} \qquad \theta_{m,n} = -\ \theta_{-m,-n}$$

$$\text{(7b)} \qquad \phi_{m,n} = -\ \phi_{-m,-n} \ .$$

The solutions represented by Eqs. (6) and (7) are chosen for a specific reason. That represented by Eqs. (5) and (6) has finite phase and zero log-amplitude in the plane $z=0$. The solution represented by Eqs. (5) and (7) have zero phase and finite log-amplitude at $z=0$. The sum of the two solutions can be used to represent any independently generated combination of log-amplitude and phase.

The procedure thus suggested is to generate two statistically independent random fields for log-amplitude and phase respectively. This procedure is especially appropriate for a plane wave propagated horizontally through the atmosphere a sufficiently long distance, $L >> 2\pi a^2/\lambda$; (a is the receiver aperture size). For such ranges the cross covariance $B_{\ell\phi}(\rho)$ is negligibly small [Tatarski, 1971, Eqs. (36)-(38), (46)]. However, the assumption of zero cross covariance for vertical paths where the effective range is limited is not so obvious.

A second technique being investigated for generating the random phase fronts involves the use of the Karhunen-Loève series [Davenport, 1958]. Here the correlation function $B(\overline{r},\overline{r}')$ is automatically built into the functions because it is the kernal of the integral equation

$$\int B(\overline{r},\overline{r}')\, f_n(\overline{r})\, d\overline{r} = \lambda_n\, f_n(\overline{r}') \tag{6}$$

used to generate $\{f_n\}$, the set of orthonormal functions. There is further only a one-dimensional set of random numbers to consider, namely the random function coefficients in the series for the phase front. One great advantage in the use of the K-L series is the combined statistical as well as functional orthogonality property of the functions.

In the Karhunen-Loève procedure the phase covariance derived from a Von Karmann index spatial spectrum was used. An 8 x 8 element array square aperture gave sufficient precision for the lower order polynomials. The procedure is quite similar to one used previously [Moreland, 1969].

The procedure for generating the set of random phase fronts is then to generate the series of random coefficients, a_n, for each member of the set using a random number generator and sum at every point in the array. Once the phase fronts have been generated, the random log-amplitude functions are generated as indicated previously.

This procedure has the advantage that only a one dimensional set of random numbers needs to be generated per array, but has the disadvantage that the series must be summed for every point in every member of every set.

To summarize, two methods of generating random phase and log-amplitude fields are being investigated. The first method generates a set of random arrays and tailors them so as to have the requisite probability distribution and covariance. The second method generates a set of eigenfunctions having the correct covariance. The series expansions for the phase and log-amplitude are then summed using random coefficients to provide one member of the set of wavefronts. We are also studying the relationship between the random arrays and atmospherically degraded log-amplitude and phase fronts to assure that the computer generated arrays represent as accurately as possible the real atmosphere.

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APPENDIX A

In this appendix the random number generation scheme mentioned in the text is described. The random number generators are based on the power residue method (IBM 6C20-8011-0). This method consists of the following steps:

1. Choose for a starting value any odd integer u_0.
2. Choose a constant multiplier of the form $x = 8t\pm3$ where t is an integer such that x is close to $2^{b/2}$ where the machine word length is b bits.
3. Compute the product xu_0. This product is 2b bits long. The higher order b bits are discarded and the lower order b bits become the next number u_1.

In this manner the random numbers are generated according to the formula

$$u_{n+1} = xu_n.$$

It is understood in the above discussion that the radix point in the u_i's is at the extreme right of the word so that the numbers are integers. After generation however the radix point is moved to the extreme left of the word (by dividing by the maximum representable integer) to produce a number between zero and unity. At this point the random numbers can be scaled to produce the required range of values, e.g., 1-64.

APPENDIX B

In this appendix we present the derivation of the expression for the spatial spectrum of the phase structure function. The situation considered has a plane wave propagating vertically downward through the earth's boundary layer where the turbulence structure constant is a function of altitude. The outer scale is also a function of altitude but this will be neglected because it has little effect on the resulting expressions. The object is to present an expression for the phase structure function spectrum normalized in a simple fashion; in this case normalized to the appropriate value for the coherence length, r_0.

The basic expressions for obtaining the phase struction function spectrum have been given [Tatarski, 1971]. Combining Eqs. (8) and (9) on p. 243 and Eq. (15a) on p. 230 and substituting for the relative permittivity spectrum $\Phi_\varepsilon(\bar{\kappa})$ in terms of the refractive index spectrum, $\Phi_n(\bar{\kappa})$, as in Eq. (B1),

(B1) $$\Phi_\varepsilon(\bar{\kappa}) = 4\Phi_n(\bar{\kappa}) = 4\ C_n^2(\bar{\eta})\ \Phi_{no}(\bar{\kappa}),$$

we have for the plane wave phase structure function spectrum

(B2a) $$F_s(\kappa) = \pi k^2\ \Phi_{no}(\kappa,0) \int_0^L \left(1 + \cos\left(\frac{\kappa^2(L-\eta)}{k}\right)\right) C_n^2(\kappa)\ d\eta\ .$$

The corresponding expressions for the phase structure function and covariance are respectively

(B2b) $$D_s(\rho) = 4\pi \int_0^\infty (1 - J_0(\kappa\rho))\ F_s(\kappa)\ \kappa d\kappa$$

(B2c) $$B_s(\rho) = 2\pi \int_0^\infty J_0\ (\kappa\rho)\ F_s(\kappa)\ \kappa d\kappa\ .$$

In Eqs. (1) and (2) η is the longitudinal variable running from transmitter to receiver and κ is the magnitude of the transverse components of spatial frequency, $\kappa = \sqrt{\kappa_x^2 + \kappa_y^2}$. ($\kappa_z$ and η are in the longitudinal direction.)

For spatial frequencies corresponding to fluctuations in the size range 5 mm to 5 m, for ranges in excess of 1 km, and for visible light we can replace the cosine term in Eq. (B2a) by its average, zero. We

further use the height variation of the index structure parameter, $C_n^2(h)$ for well developed turbulence in unstable air [Wyngaard, 1971]

$$\text{(B3)} \qquad C_n^2(h) = C_n^2(h_o)\left(\frac{h}{h_o}\right)^{-4/3} .$$

Thus, taking the source at a very large height and receiver near ground level we have

$$\text{(B4)} \qquad F_s(\kappa) \doteq \pi k^2\, \Phi_{no}(\kappa,0) \int_0^L C_n^2\,(H_o)\left(\frac{H_o+L-\eta}{H_o}\right)^{-4/3} d\eta$$

where H_o is the height of the receiver and $C_n^2(H_o)$ is the structure parameter value at the receiver height, (η=L). Performing the η integration and substituting for $\Phi_{no}(\kappa,0)$ using the Von Karmann spectrum we have

$$\text{(B5)} \qquad F_s(\kappa) = \pi k^2 \cdot 3H_o C_n^2(H_o)\left(1-\left(1+\frac{L}{H_o}\right)^{-1/3}\right) x .033\left\{\left(\frac{1.077}{L_o}\right)^2 + \kappa^2\right\}^{-11/6} .$$

Taking the source much higher than the receiver, $L + H_o \gg H_o$ and rearranging, we have

$$\text{(B6)} \qquad F_s(\kappa) = .033\pi k^2\,(3H_o)C_n^2(H_o)\left\{\left(\frac{1.077}{L_o}\right)^2 + \kappa^2\right\}^{-11/6}$$

where L_o is the outer scale and the factor, 1.077, is included to make the asymptotic value of the index structure for large separation, $D_n(\infty) \doteq C_n^2 L_o^{2/3}$. This allows convenient determination of L_o from a log-log plot of D_n versus ρ: the break point occurs where the separation equals L_o.

Equation (6) is the desired spectrum. However it will be more convenient to express it using the coherence length, r_o, for normalization. For this we find the expression for the wave structure function appropriate to this situation. The expression for the wave structure function spectrum is obtained from the phase structure function spectrum by putting the cosine term to unity in Eq. (B2a);

$$F_W(\kappa) = 2\pi^2 \, \Phi_{no}(\kappa,0) \int_0^L C_n^2(\eta) \, d\eta \, . \tag{B7}$$

Performing the longitudinal integration gives

$$F_W(\kappa) = .033\pi k^2(3H_o) \, C_n^2(H_o) \left\{ \left(\frac{1.077}{L_o}\right)^2 + \kappa^2 \right\}^{-11/6} \, . \tag{B8}$$

In the inertial subrange, $\kappa^2 >> (1.077/L_o)^2$, the associated structure function is

$$D_W(\rho) = 2 \times .033\pi k^2(3H_o)C_n^2(H_o) \times 4\pi \int_0^{\infty} (1-J_o(\kappa\rho))\kappa^{-8/3} \, d\kappa \tag{B9a}$$

$$= \frac{4 \times .033\pi^2 \times (6/5)\Gamma(1/6)}{\Gamma(11/6) \, 2^{5/3}} \, k^2(3H_o)C_n^2(H_o)\rho^{5/3} \tag{B9b}$$

$$= 6.88(\rho/r_o)^{5/3} \, . \tag{B10}$$

Combining Eqs. (B9b) and (B10) we have

$$r_o = (6.88/2.91k^2(3H_o)C_n^2(H_o))^{3/5} \, . \tag{B11}$$

It is interesting to note that the value for r_o thus obtained corresponds to the plane wave value for horizontal propagation at the height of the receiver and over a path length of only three times the receiver height.

Substituting for $k^2(3H_o)C_n^2(H_o)$ in Eq. (B8) gives the desired normalized form of the phase structure spectrum.

$$F_s(\kappa) = .245 \, r_o^{-5/3} \left\{ (1.077/L_o)^2 + \kappa^2 \right\}^{-11/6} \, . \tag{B12}$$

For many purposes the receiver is much smaller than the outer scale, (the outer scale being of the same order of magnitude as the height). For such cases the outer scale can be neglected, giving

$$F_s(\kappa) = .245\, r_c^{-5/3}\, \kappa^{-11/3} \tag{B13}$$

Equations (12) and (13) are the main results of this appendix.